Pergamon

Int. J. Heat Mass Transfer. Vol. 40, No. 7, pp. 1713–1716, 1997

0017-9310/97 $17.00+0.00

PII: S0017-9310(96)00228-1

TECHNICAL NOTES

The interface temperature of two suddenly contacting bodies, one of them undergoing phase change

T. LOULOU and D. DELAUNAY

Laboratoire de Thermocinétique, URA CNRS 869, La Chantrerie, C.P. 3023, 44087 Nantes Cedex, France

(*Received* 14 *March* 1995 *and in final form* 24 *June* 1996)

INTRODUCTION

The interface temperature of two suddenly contacting semi-infinite bodies is commonly obtained from the analytical solution of the heat equation. In the classical case no phase change has been considered in any body. However, phase change in one of the contacting bodies is often encountered in many practical applications, such as metal casting, injection molding, welding, etc. No solution for this problem can be found in the literature. The classical solution of two suddenly contacting solid bodies, initially at different, but uniform temperatures, under the assumption that no thermal contact resistance exists between them, is a constant interface temperature. This average temperature shares the difference between the initial temperatures in the inverse thermal effusivities ratio. The purpose of this study is to determine the interface temperature of two semi-infinite bodies, suddenly coming into contact, in the case where one of them undergoes phase change. The two bodies are held initially at two uniform, but different temperatures. The initial temperature of the liquid body is supposed higher than its melting temperature. We consider constant thermal physical properties, so the linear form of the heat conduction equation is adopted in this analysis.

THERMAL ANALYSIS

We consider the case of two semi-infinite bodies with constant, but different physical properties. The first body (1) is a solid medium and its initial temperature is T_1^i. The second body (2) is a liquid medium and its initial and melting temperature are respectively T_2^i and T_2^m. We suppose that there is no difference between the solid and liquid density of body (2), so $\rho_s = \rho_l$. No convective effects are considered in the liquid phase.

At time $t > 0$, the bodies are placed in perfect contact, Fig. 1. The contacting plane is supposed at $x = 0$. After the contact, two cases may be considered.

The interface temperature $\mathrm{T_2(0, t)}$ *is greater than* $\mathrm{T_2^m}$

The second medium remains liquid and for pure conduction the solution of such a problem is classical. The temperature fields in the two media have the following expressions, [1]:

$$T_1(x,t) = \frac{T_1^i b_1 + T_2^i b_2}{b_1+b_2} + \frac{b_2(T_1^i - T_2^i)}{b_1+b_2}\,\mathrm{erf}\left(\frac{-x}{2\sqrt{\alpha_1 t}}\right)$$

$$= T_{\mathrm{int}} - (T_{\mathrm{int}} - T_1^i)\,\mathrm{erf}\left(\frac{-x}{2\sqrt{\alpha_1 t}}\right) \quad (1)$$

for medium (1), and

$$T_2(x,t) = \frac{T_1^i - {}_2^{\mathrm{Ti}} b_2}{b_1+b_2} + \frac{b_1(T_2^i - T_1^i)}{b_1+b_2}\,\mathrm{erf}\left(\frac{x}{2\sqrt{\alpha_2 t}}\right)$$

$$= T_{\mathrm{int}} - (T_{\mathrm{int}} - T_2^i)\,\mathrm{erf}\left(\frac{x}{2\sqrt{\alpha_2 t}}\right) \quad (2)$$

for medium (2).

In this case, the interface temperature, at the contacting surface $x = 0$, is constant and is given by:

$$T_{\mathrm{int}} = \frac{T_1^i b_1 + T_2^i b_2}{b_1+b_2} \quad (3)$$

where $b = \sqrt{\rho_i \lambda_i C p_i}$, $i = 1, 2$ is the effusivity of the corresponding bodies.

This result, valid immediately after the contact ($t \to 0^+$) shows the establishment of an average temperature at the contact surface which remains constant during the total time of contact.

The heat fluxes at the interface $x = 0$ have the following form:

$$\varphi_1(0,t) = \frac{b_1}{\sqrt{\pi t}}(T_1^i - T_{\mathrm{int}}) \quad (4)$$

$$\varphi_2(0,t) = \frac{b_2}{\sqrt{\pi t}}(T_{\mathrm{int}} - T_2^i) \quad (5)$$

The equality between these two expressions verifies the hypothesis of a constant interface temperature given in formula (3).

The interface temperature $\mathrm{T_2(0, t)}$ *is smaller than* $\mathrm{T_2^m}$

In this case, the problem is quite different. Under the assumption that $T_2(0, t)$ remains smaller than T_2^m, region (2) will solidify. The solidification begins at the boundary surface $x = 0$ and the solid–liquid interface, $S(t)$, moves in the positive $\vec{ox}$ direction. The problem, in body (2), is the solidification of a semi-infinite slab.

The mathematical formulation of such a problem, in the general case of perfect contact, can be written as:

$$\frac{\partial T_1(x,t)}{\partial t} = \alpha_1 \frac{\partial^2 T_1(x,t)}{\partial x^2}, \quad -\infty < x < 0, \quad t > 0 \quad (6)$$

$$\frac{\partial T_2^s(x,t)}{\partial t} = \alpha_2^s \frac{\partial^2 T_2^s(x,t)}{\partial x^2} \quad 0 < x < S(t), \quad t > 0 \quad (7)$$

NOMENCLATURE

b	thermal effusivity
C_p	heat capacity
L	latent heat
T	temperature
t	time
x	space.

Greek symbols

α	thermal diffusivity
λ	thermal conductivity
ρ	density
φ	heat flux
ξ	constant.

Subscripts

1	body 1
2	body 2
i	= 1, 2 representing bodies 1 and 2 respectively
int	interface
f	fluid.

Superscripts

i	initial
l	liquid
m	melting
s	solid.

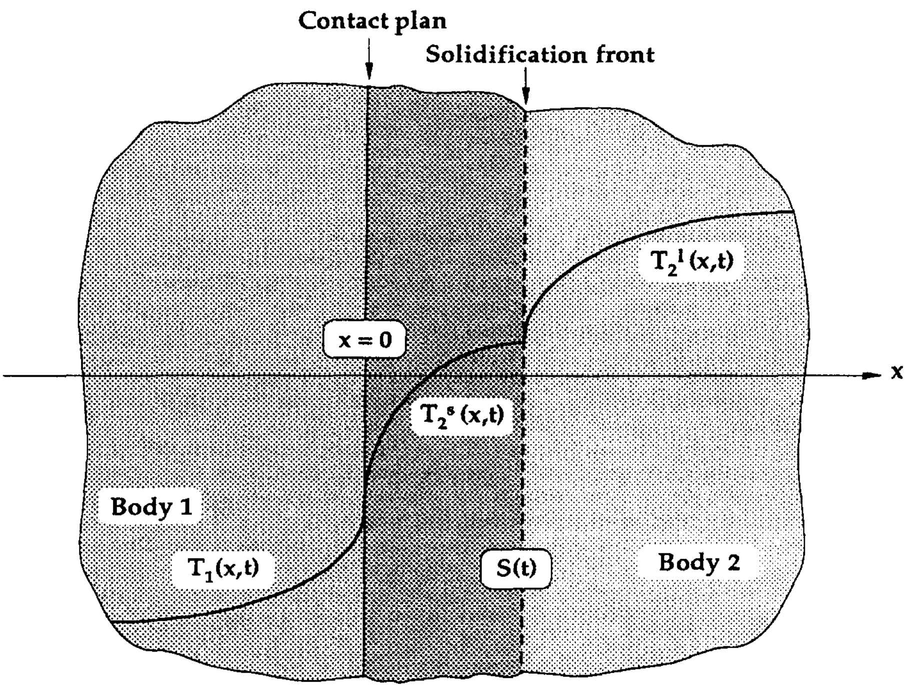

Fig. 1. Geometry and coordinates.

$$\frac{\partial T_2^l(x,t)}{\partial t} = \alpha_2^l \frac{\partial^2 T_2^l(x,t)}{\partial x^2} \quad S(t) < x < +\infty, \quad t > 0 \tag{8}$$

$$\lambda_1 \frac{\partial T_1(x,t)}{\partial x} = \lambda_2^s \frac{\partial T_2^s(x,t)}{\partial x}, \quad x = 0, \quad t > 0 \tag{9}$$

$$T_1(x,t) = T_2^s(x,t), \quad x = 0, \quad t > 0 \tag{10}$$

$$\lambda_2^s \frac{\partial T_2^s(x,t)}{\partial x} - \lambda_2^l \frac{\partial T_2^l(x,t)}{\partial x} = \rho_2 L \frac{\mathrm{d}S(t)}{\mathrm{d}t}, \quad x = S(t), \quad T > 0 \tag{11}$$

$$T_2^s(x,t) = T_2^l(x,t) = T_2^m, \quad x = S(t), \quad t > 0 \tag{12}$$

$$T_1(x,t) = T_1^i, \quad x \to -\infty, \quad t > 0 \tag{13}$$

$$T_2(x,t) = T_2^i, \quad x \to +\infty, \quad t > 0 \tag{14}$$

$$T_1(x,t) = T_1^i, \quad -\infty < x \leqslant 0, \quad t = 0 \tag{15}$$

$$T_2(x,t) = T_2^i, \quad 0 \leqslant x < +\infty, \quad t = 0. \tag{16}$$

In the classical case, without phase change, formulas (1) and (2) may be established by considering that the interface temperature does not depend on time. Then by considering each medium as a semi-infinite body submitted to a temperature step (respectively, $T_{int} - T_1^i$ and $T_{int} - T_2^i$ for mediums (1) and (2)), the heat fluxes equality on the interface allows one to calculate T_{int}.

The same approach was used when one of the media undergoes solidification. To attempt a decoupling between the two media, a very strong hypothesis is tried: the interface temperature $T_1(0, t)$ (or $T_2(0, t)$) remains constant and equal to a value T_{int}. Equation (10) is then verified. If the equality of the heat fluxes on the contacting surface between medium (1) and (2) allows one to compute a value of T_{int} which does not depend on the time t, as for the classical case, the hypothesis will be valid. The problem in the three regions: body (1), solid body (2), liquid body (2) can be separated into two problems. The first as a semi-infinite solid body $-\infty < x \leqslant 0$ with a step at $x = 0$. The second problem is the solidification in half-space with a temperature step at $x = 0$. The solution of the first problem is given in formula (1) and then verifies equations (6), (10), (13) and (15). The solution of the second problem can be found in refs. [1–3]. The temperature of the solid, in region (2), is given by:

$$T_2^s(x, t) = T_{int} + \frac{T_2^m - T_{int}}{\operatorname{erf}(\xi)} \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha_2^s t}}\right). \quad (17)$$

The temperature of the liquid phase is given by:

$$T_2^l(x, t) = T_2^i + (T_2^m - T_2^i) \frac{\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha_2^l t}}\right)}{\operatorname{erfc}\left(\xi\sqrt{\frac{\alpha_2^s}{\alpha_2^l}}\right)} \quad (18)$$

where l and s refer respectively liquid and solid phases of medium (2). These expressions verify equations (7), (8), (12), (14) and (16).

The parameter ξ is determined from the solution of the following transcendental equation:

$$\frac{\exp(-\xi^2)}{\operatorname{erf}(\xi)} + \frac{\lambda_2^l}{\lambda_2^s}\sqrt{\frac{\alpha_2^l}{\alpha_2^s}} \frac{T_2^m - T_2^i}{T_2^m - T_{int}} \times \frac{\exp\left(-\xi^2 \frac{\alpha_2^s}{\alpha_2^l}\right)}{\operatorname{erfc}\left(\xi\sqrt{\frac{\alpha_1^s}{\alpha_1^l}}\right)} = \frac{\xi L_2 \sqrt{\pi}}{Cp_2^s(T_2^m - T_{int})} \quad (19)$$

obtained by equation (11) and the relation $S(t) = 2\xi\sqrt{\alpha_2^s t}$. The heat flux in the solid phase and at the interface $x = 0$ is equal to:

$$\varphi_2^s(0, t) = \frac{b_2^s}{\sqrt{\pi t}} \frac{(T_{int} - T_2^m)}{\operatorname{erf}(\xi)} \quad (20)$$

The equality between the heat fluxes at the interface $x = 0$ given in equations (4) and (20), allows one to verify equation (9) and gives the interface temperature:

$$T_{int} = \frac{T_2^m b_2^s + T_1^i b_1 \operatorname{erf}(\xi)}{b_2^s + b_1 \operatorname{erf}(\xi)}. \quad (21)$$

We see that T_{int} remains constant and validates our strong hypothesis. The new instantaneous interface temperature depends on ξ which itself depends on the interface temperature T_{int}. In such a situation we use an iterative process to find the solution. To start the iterative procedure, we arbitrarily guess the interface temperature and we compute the corresponding ξ from equation (19). This is used to compute the new interface temperature in formula (21). The algorithm converges very quickly.

EXAMPLES

To illustrate the above analysis, numerical tests were performed to estimate the interfacial temperature. These simulations are done in two different cases. The first one is the sudden contact between molten tin and a solid substrate of nickel. The second case concerns injection molding (polymer–steel). The last case concerns the molding of molten polymer on a solid one (overmolding). The values of physical properties used in this analysis, and taken from refs. [3, 4], are given in Table 1. Results of these examples are given in Table 2.

CONCLUDING REMARKS AND COMMENTS

Formula (21) is quite different from the classical one used for thermal contact without phase change. The numerical examples show that for contacting materials with effusivities of the same order of magnitude, the results differ strongly from the solution without phase change. This is very important for morphological considerations. However, it must be noted that this approach is purely theoretical and exact from the mathematical point of view. In realistic situations this simple approach must be complicated by taking into account thermal contact resistance, supercooling effects, if necessary, and other complex phenomenon. In any case the use of the exact formula is an improvement in comparison with the one which is actually used.

Table 1. Value of the properties used in the simulations

Property	Tin	Nickel	Polymer	Steel
ρ [kg m^{-3}]	7300.0	8900.0	910.0	7850.0
C_p [J kg^{-1} K^{-1}]	226.0	450.0	1910.0	460.0
λ [W m^{-1} K^{-1}]	60.0	90.0	0.43	30.0
L [J kg^{-1}]	59 000.0	—	54 000.0	—
Melting temperature [K]	505.0	—	453.0	—

Table 2. Results

<table>
<tr><th></th><th>Tin</th><th>Nickel</th><th>Polymer</th><th>Steel</th><th>Polymer</th><th>Polymer</th></tr>
<tr><td>Initial temperature [°C]</td><td>300.0</td><td>20.0</td><td>230.0</td><td>40.0</td><td>230.0</td><td>40.0</td></tr>
<tr><td>Interface temperature [°C]</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Formula (3)</td><td colspan="2">116.0</td><td colspan="2">54.0</td><td colspan="2">135.0</td></tr>
<tr><td>Formula (21)</td><td colspan="2">150.0</td><td colspan="2">56.0</td><td colspan="2">142.0</td></tr>
</table>

REFERENCES

1. Carslaw, H. S. and Jaeger, J. C., *Conduction of Heat in Solids*, 2nd edn. Oxford University Press, London, U.K., 1959, pp. 50–91.
2. Cranck, J., *Free and Moving Boundary Problems*. Oxford University Press, London, U.K., 1984, pp. 101–103.
3. Özisik, M. N., *Heat Conduction*. Wiley, New York, 1980, pp. 397–434.
4. Manceau, I., Heat transfer modelling during injection molding. Ph.D. Thesis, University of Nantes, Nantes, 1994.

Pergamon

Int. J. Heat Mass Transfer. Vol. 40, No. 7, pp. 1716–1719, 1997

0017–9310/97 $17.00+0.00

PII: S0017–9310(96)00220–7

The unsteady solutions of a unified heat conduction equation

CHAUR-KIE LIN, CHI-CHUAN HWANG† and YEON-PUN CHANG

Department of Mechanical Engineering, Chung Yuan University, Chung Li, Taiwan 32023, Republic of China

(*Received* 8 *January* 1996)

1. INTRODUCTION

The short-phase laser heating process of metals is composed of three general steps: the deposition of radiation energy on electrons, the transport of energy by electrons and the propagation of energy through media. The propagation of energy during a relatively slow heating process can be modeled by the Fourier heat conduction model, since the deposition of radiation energy can be assumed to be instantaneous. However, it takes time, in reality, to establish an equilibrium state in thermodynamic transition. For a problem involving reflectivity change resulting from short-pulse laser heating on gold films [1], the response time is on the order of picoseconds, comparable to the time required to establish an equilibrium state. The diffusion theory fails under such circumstances because the hot electron gas and the metal lattice cannot reach thermodynamic equilibrium in such a short period of time. Thus, more general and rigorous models are needed to include effects of electron-lattice interactions and non-Fourier transport. After Maxwell's research [2] on the kinetic theory of gases, which has had great influence on the development of the thermal wave theory, modifications on Fourier's law are promoted by its deficiencies in advanced applications [3–16].

The unified model (Tzou [16]) is a generalized approach based on the dual-phase-lag concept which accounts for the lagging behavior in the high-rate response. A universal constitutive equation between the heat flux vector and the temperature gradient is proposed with an effort to cover a wide range of physical responses from microscopic to macroscopic scales in both space and time [16]. An exact solution, using the method of separation of variables, to the above universal constitutive equation for a one-dimensional problem is addressed in this paper. Part of the results are found to be different from those by Tzou [16]. The aim of this note is to present a convenient approach to the short-pulse laser heating problem by virtue of the unified heat conduction equation.

2. THEORETICAL ANALYSIS

The short-pulse laser heating of a metal film can be treated as a one-dimensional problem because its heat penetration depth is much smaller than the beam diameter. The solid is assumed to have a finite thickness, l. The phase lag of the heat flux and that of the temperature gradient are τ_q and τ_T, respectively. An initial temperature distribution of constant value, T_0, in solid and an imposed initial time-rate change of temperature, $\dot{T}_0$, are given. A suddenly-raised temperature T_w at left end $x \times 0$ and a zero temperature gradient remaining at right end $x = l$ are suitable boundary conditions for this type of problem. After introducing the following dimensionless variables as in ref. [16],

$$\theta = \frac{T-T_0}{T_w - T_0}, \quad \delta = \frac{x}{l}, \quad \text{and} \quad \beta = \frac{t}{(l^2/\alpha)} \tag{1}$$

the temperature field equation, the initial conditions and the boundary conditions become:

$$\frac{\partial^2 \theta}{\partial \delta^2} + z_T \frac{\partial^3 \theta}{\partial \delta^2 \partial \beta} = \frac{\partial \theta}{\partial \beta} + z_q \frac{\partial^2 \theta}{\partial \beta^2} \tag{2}$$

and

$$\theta = 0 \quad \text{and} \quad \frac{\partial \theta}{\partial \beta} = \dot{\theta}_0 \quad \text{at } \beta = 0 \tag{3}$$

$$\theta = 1 \quad \text{at } \delta = 0 \quad \text{and} \quad \frac{\partial \theta}{\partial \delta} = 0 \quad \text{at } \delta = 1 \tag{4}$$

† Author to whom correspondence should be addressed.